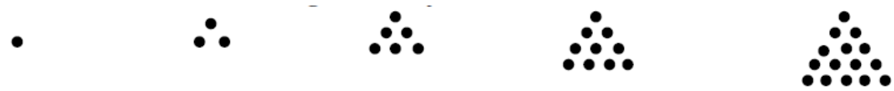
Grade 9 Mathematics Worksheet

Number Patterns

Questions:

1. Consider the array of dots in the following diagram:



These numbers are known as *the triangular numbers*.

a) For this sequence of numbers, complete the table below:

Picture number	1	2	3	4	5	6	9
Number of dots	1	3	6				

b) Show clearly how you got to T_9 in this sequence.

c) The general rule for the triangular number sequence 1 ; 3; 6; ... is $T_n = \frac{n(n+1)}{2}$.

If the sequence started at 3 and continued as normal, show how you will adjust this rule to accommodate this change?

d) Use the rule $T_n = \frac{n(n+1)}{2}$ to prove that two consecutive triangular numbers always add up to a square number.

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Solution:

1. a)

Picture number	1	2	3	4	5	6	9
Number of dots	1	3	6	10	15	21	28

b) $T_2 = T_1 + 2$;
$T_3 = T_2 + 3$;
$T_4 = T_3 + 4$;
$T_5 = T_4 + 5$;
$T_6 = T_5 + 6$;
$T_7 = T_6 + 7$; so $T_9 = T_8 + 9$ or in general the recursive pattern is $T_{n+1} = T_n + (n+1)$.

c) This will be a horizontal translation of 1 unit to the left for the sequence. It thus becomes:

Picture number	1	2	3
Number of dots	3	6	10

d) $T_n = \frac{(n+1)(n+1+1)}{2} = \frac{(n+1)(n+2)}{2}$.

For two consecutive numbers that are triangular:

$T_n = \frac{n(n+1)}{2}$ and $T_{n+1} = \frac{(n+1)(n+2)}{2}$.

So:

$$\begin{aligned} T_n + T_{n+1} &= \frac{n(n+1)}{2} + \frac{(n+1)(n+2)}{2} \\ &= \frac{(n+1)(n+n+2)}{2} \\ &= \frac{(n+1)(2n+2)}{2} \\ &= \frac{2(n+1)(n+1)}{2} \\ &= (n+1)^2 \end{aligned}$$

This is clearly a square number.

Learners learn about sequences by looking at them recursively – that is what happens to the previous term to obtain the next term.

The translation is one unit to the left for the sequence, so that term 1 is now 3. Thus we need to change the n in the sequence to $n + 1$ to accommodate this translation. Another way to look at this is realise that n is still one and that what we change in the generalised number must have an output of 3.